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Transformation of Whittaker function of three variables

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Abstract: The main object of the present paper is to obtain transformation of Whittaker functions of three variables into Srivastava's triple hypergeometric series $F^{(3)}$. A number of known and new transformations for Kampe$'$ de Fe$'$riet function, Appell function F_2 are also obtained as special cases.

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1. Introduction and Definition

Whittaker functions $M_{\lambda,\mu}(x)$ and $W_{\lambda,\mu}(x)$ are solutions of the Whittaker equation

$$W'' + \left(\frac{\frac{1}{4}-\mu^2}{x^2} + \frac{\lambda}{x} - \frac{1}{4}\right) w = 0, \tag{1.1}$$

the function $W_{\lambda,\mu}(x)$ satisfies the equation

$$W_{\lambda.\mu}(x) = \frac{\Gamma(-2\mu)}{\Gamma(\frac{1}{2}-\lambda-\mu)}\ M_{\lambda,\mu}(x) + \frac{\Gamma(2\mu)}{\Gamma(\frac{1}{2}-\lambda+\mu)}\ M_{\lambda,-\mu}(x). \tag{1.2}$$

The pair of functions $M_{\lambda,\mu}(x)$, $M_{\lambda,-\mu}(x)$ and $W_{\lambda,\mu}(x)$ and $W_{\lambda,-\mu}(x)$ are linearly independent solutions of equation (1.1). Whittaker function $M_{\lambda,\mu}(x)$, was introduced by Whittaker [11] (see also Whittaker and Watson [12]) in terms of Confluent hypergeometric function ${}_1F_1$ (or Kummer's function).

$$M_{\lambda,\mu}(x) = x^{\mu+\frac{1}{2}} e^{-\frac{1}{2}x}\ {}_1F_1\left(\mu-\lambda+\frac{1}{2}, 2\mu+1; x\right), \tag{1.3}$$

$$W_{\lambda,\mu}(x) = x^{\mu+\frac{1}{2}} e^{-\frac{1}{2}x}\ \Psi\left(\mu-\lambda+\frac{1}{2}, 2\mu+1; x\right). \tag{1.4}$$